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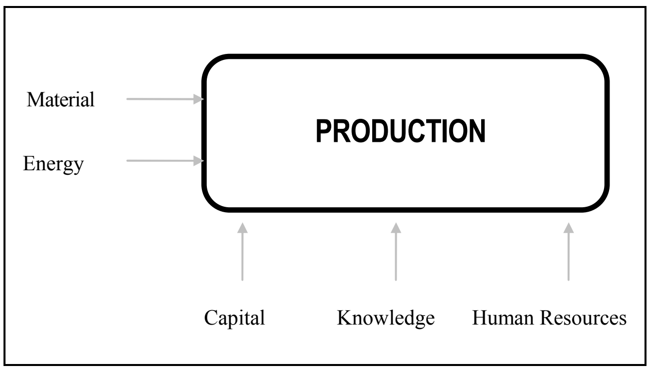
The Mathematical Analysis of Macro-economics and the Dynamic Model of Production



The Dynamic model of production regards production as a function with five variables: Capital (equipment, artifacts) represented by **K**, Human resources represented by **L**, Knowledge (information) represented by **I**, Material represented by **M**, and Energy represented by **E**; over Time represented by **t**. The mathematical presentation of the above model is as following:

$$Q = f(K, L, I, M, E, t)$$

Should one assume that in a given process, the consumed material and energy (M and E) are always linearly proportional to the final products (Q), (e.g. waste of material and energy is always constant), the above formula can be simplified to:

$$Q = f(K, L, I, t)$$

If we assume that Knowledge (information) is not an element of production, **(<u>and that is a big IF</u>)** then the above relationship can be simplified to:

$$Q = f(K, L, t)$$

Orthodox mathematical analysis of macroeconomics, being based on $Q = f(K, L, t)$, is consistent with the above assumptions: *knowledge is not a variable, material and energy are always proportional to final products*. Sundry economics textbooks have already confined the mathematical analysis of macroeconomics to two variables: capital and labor; which is consistent with the very simplified formula of: $Q = f(K, L)$. The following citations reflect this point of view:

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If Q represents output and K and L represent capital and labor inputs in 'physical' units, then the aggregate production function can be written as

$$Q = F(K, L; t)$$

The variable t for time appears in F to allow for technical change. It will be seen that I am using the phrase 'technical change' as a shorthand expression for *any kind of shift* in the production function. Thus slowdowns, speedups, improvements in the education of the labor force, and all sort of things will appear as 'technical change'.

Solow R, 1957, **Technical Change and the Aggregate Production Function,** Review of Economics and Statistics, August 1957, pp. 312-20. Reprint in "Rosenberg N, 1971, **The Economics of Technical Change**, Pelican, London).

The simplest, and so most widely used, orthodox economic theory employs the following aggregate production function:

$$Q = f(K, L, A, t) \qquad (3.1)$$

Here, Q is total real output or income, K is the total real capital stock, L is the total labor force employed, A is the total land used, and t is time. Q, K, L, and A must all be interpreted as some sort of index numbers which combine heterogeneous quantities by suitable weights in the manner just described. Increase in K, L, A, or t each increase Q. The increase that results purely from the passage of time, K, L, and A being constant, is due to technical progress.

Production function theory, for the most part, assumes that the function (3.1) is subject to constant returns to scale, so that if the quantities of capital and labor (ignoring land) were both to increase in the same proportion, say 1 percent, then output would also increase in that proportion. This is in the absence of technical progress, and is based on the idea of reduplication. Since K and L are homogeneous, 1 per cent more K plus 1 per cent more L yields 1 per cent more of every useful input that is already there, and should, one would think, produce 1 per cent more output.

Production function further assumes diminishing returns to the stocks of each factor. This means that, for example, the higher is K/L, with given technology, the lower is dQ/dK, the marginal product of capital, and the higher is dQ/dL, the marginal product of labor.

The second explanation for the same phenomenon is that the production function complies with the Cobb-Douglas relationships as follows:

$$Q = Be^{mt}K^{a}L^{1-a} \qquad (3.2)$$

B is a constant depending on choice of units, and the term e^{mt} shows how technical progress expands output at the rate m when K and L are both constant.

Scott, 1989, **A New View of Economic Growth**, Paul S. Strassmann refers to the Cobb-Douglas equation's deficiency and its role in "computer paradox".

The Cobb-Douglas production function equation is a favorite tool of econometric academics. This equation considers capital and labor inputs as the determinants of all outputs. It is this adherence to industrial age thinking that assumes that only a small number of inputs factors, particularly capital, explain the generation of productivity grains. … Nowadays information and knowledge are a significant component in the costs of goods and services, but they do not fit Cobb-Douglas.

Paul S. Strassmann, 1977, **The Squandered Computer: Evaluating the Business Alignment of Information Technologies**, The Information Economics Press, New Canaan, Connecticut

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[1] Last revised: September 24, 1999